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Brief communication

On the effect of anisotropy on the turbulent dispersion and deposition of small particles

Yi Wang, P.W. James *

School of Mathematics and Statistics, University of Plymouth, Drake Circus, Plymouth, PL4 8AA, UK

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1. Introduction

A widely used method for the prediction of particle dispersion in turbulent flows is based on the eddy interaction model, or EIM, (see, for example, Gosman and Ioannides, 1981; or Shuen et al., 1984). In this model, the instantaneous flow field is reconstructed from mean flow quantities by assuming that it is comprised of eddies whose lifetimes and length scales can be deduced from local mean flow quantities. Usually, a turbulence model of the k–ϵ type is used to provide values of the mean flow quantities and, as a consequence, the reconstructed fluctuating part of the flow field is isotropic. This feature is a deficiency of the model which becomes significant in regions where the turbulence structure is anisotropic. The deficiency can be overcome by more sophisticated models which use the information provided by Reynolds-stress turbulence models (see, for example, Berlemont et al., 1990; Zhou and Leschziner, 1991; 1996; and Burry and Bergeles, 1993).

However, k–ϵ models are still widely used in industrial flow problems to predict mean flow quantities in regions where the flow is anisotropic. It is, therefore, the aim of this paper to provide a simple modification of the EIM, based on damping functions, which can account for some of the effects of anisotropy in near-wall regions within the framework of k–ϵ type turbulence models. The intention is to use the method to simulate droplet dispersion in wave–plate demisters, in which low intensity turbulent flows typically occur. However, it is hoped that the proposed modification to the EIM will have wider application to industrial flows in which near-wall anisotropy is important.

The damping functions are obtained from the results obtained by Kim et al. (1987) and Mansour et al. (1988) for the direct numerical simulation (DNS) of turbulent flow through a duct at a Reynolds number, based on the duct half width and friction velocity, of 180. The method is then used to predict particle dispersion in a channel and the results are compared with those of Kallio and Reeks (1989) and Liu and Agarwal (1974).

* Corresponding author.

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2. The eddy interaction model and its modification

In the EIM, the trajectories of a large number of particles are simulated individually. For liquid droplets (modelled as hard spheres) moving through air, the gas to particle density ratio is very small and this allows the particle equation of motion to be greatly simplified. The particle-laden flow is assumed to be dilute so that particle–particle interaction and the effect of particles on the flow field may be neglected. The influence of gravity is also neglected because, for the range of particle sizes to be considered, deposition is not strongly influenced by gravitational settling. Kallio and Reeks (1989) make a similar assumption when comparing their predictions with the data of Liu and Agarwal (1974). Also, it is intended to apply the results to horizontal flow through wave–plate demisters with vertical plates, in which case deposition due to gravitational settling is not the dominant mechanism. The particle equation of motion is then

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}=\frac{\mathbf{u}-\mathbf{v}}{\tau}. \tag{1}$$

In Eq. (1) $\mathbf{u}$ and $\mathbf{v}$ are the velocities of the fluid and the particle, respectively, and t is time. The relaxation time of a particle, τ, is defined by

$$\tau=\frac{4d\rho_{\mathrm{p}}}{3\rho_{\mathrm{f}}C_{\mathrm{D}}|\mathbf{u}-\mathbf{v}|} \tag{2}$$

where d is the particle diameter, $C_{\mathrm{D}}(\equiv 24/Re_{\mathrm{p}})$ is the Stokes drag coefficient, ρ_{f} and ρ_{p} are the densities of the fluid and particle, respectively, and $Re_{\mathrm{p}}=\rho_{\mathrm{f}}\ \mathrm{d}|\mathbf{u}-\mathbf{v}|/\mu$, is the particle Reynolds number in which μ is the fluid dynamic viscosity. From the assumption of Stokes drag law it follows that τ is a constant. For convenience, Eq. (1) can be non-dimensionalized by using wall variables, the fluid kinematic viscosity $\nu\equiv\mu/\rho_{\mathrm{f}}$ and the friction velocity u_τ. The result is

$$\frac{\mathrm{d}\mathbf{v}^+}{\mathrm{d}t^+}=\frac{\mathbf{u}^+-\mathbf{v}^+}{\tau^+} \tag{3}$$

where $t^+=tu_\tau^2/\nu$, $\mathbf{u}^+=\mathbf{u}/u_\tau$, $\mathbf{v}^+=\mathbf{v}/u_\tau$ and

$$\tau^+=\frac{d^2u_\tau^2(\rho_{\mathrm{p}}/\rho_{\mathrm{f}})}{\nu^2}. \tag{4}$$

Following Graham and James (1996), it is assumed that the fluid velocity remains constant during a small time step δt^+. The fluid velocity is changed whenever a particle enters a new computational control volume or when it interacts with a new eddy. The integration time step δt^+ is adjusted to allow for this. Since τ^+ is constant, Eq. (3) can be integrated exactly to find

$$\mathbf{v}^{+}(t^{+}+\delta t^{+}) = \mathbf{u}^{+} - [\mathbf{u}^{+} - \mathbf{v}^{+}(t^{+})]\exp(-\delta t^{+}/\tau^{+}). \tag{5}$$

The dimensionless position $\mathbf{x}^{+} = (x^{+}, y^{+}, z^{+})$ of a particle at time $t^{+} + \delta t^{+}$ can then also be found exactly:

$$\mathbf{x}^{+}(t^{+}+\delta t^{+}) = \mathbf{x}^{+}(t^{+}) - \tau^{+}[\mathbf{u}^{+} - \mathbf{v}^{+}(t^{+})][1-\exp(-\delta t^{+}/\tau^{+})] + \mathbf{u}^{+}\delta t^{+}. \tag{6}$$

Here $(x^{+}, y^{+}, z^{+}) = (x, y, z)u_{\tau}/v$, where (x, y, z) is the dimensional particle position. The above equations will be applied to a duct flow in which x is in the streamwise direction, y in the direction normal to the duct wall and z in the spanwise direction. The integration scheme derived above is simple to implement and is computationally efficient. In the simulations, the particles are distributed uniformly at the inlet to the flow domain and have the same velocity as the fluid.

In Eq. (3), $\mathbf{u}^{+}$ is the instantaneous flow velocity which, in the EIM, is found by adding a random fluctuating component to the mean velocity $\mathbf{U}^{+}$, i.e.

$$\mathbf{u}^{+} = \mathbf{U}^{+} + \mathbf{u}^{+'}Nr \tag{7}$$

where Nr is a random number drawn from a normal probability distribution with zero mean and unit standard deviation, and $\mathbf{u}^{+\prime}$ $\{=(u_x^{+\prime}, u_y^{+\prime}, u_z^{+\prime})\}$ is the rms value of the eddy fluctuating velocity, which is assumed to be given in terms of the turbulence kinetic energy k by

$$u_x^{+'} = u_y^{+'} = u_z^{+'} = (2k/3)^{1/2}. \tag{8}$$

It is noted that use of Eq. (8) implies that turbulence influences the construction of the eddy velocity isotropically. As pointed out by Kallio and Reeks (1989), this assumption is inappropriate when particles move in the strongly anisotropic turbulent buffer region near a wall. Kim et al. (1987) used DNS to obtain the variation of turbulent flow quantities, including rms velocities, with distance from the wall in rectangular duct flow. The data reveal that the differences between fluctuating velocity components are very large in the region $y^{+} < 30$ and that the normal component $u_y^{+\prime}$ is far smaller than the other two. Eq. (8) significantly over-estimates the value of the normal component. It will be seen later that this over-estimate leads to large discrepancies in the results for particle dispersion.

In order to overcome this problem, three functions, f_{u}, f_{v}, f_{w}, are introduced where f_{u} and f_{v} are the ratios of the streamwise and the normal velocity components to their values determined by the standard EIM, i.e.

$$f_{\mathrm{u}} = (\overline{u'_x u'_x})^{1/2}/(2k/3)^{1/2}, f_{\mathrm{v}} = (\overline{u'_y u'_y})^{1/2}/(2k/3)^{1/2}, \tag{9}$$

and f_w is found from

$$f_w = \sqrt{(3 - f_u^2 - f_v^2)}. \tag{10}$$

By curve-fitting the DNS data of Kim et al. (1987), the functions f_u, and f_v can be expressed as

$$\begin{aligned} f_u &= 1 + 0.285(y^+ + 6)\exp[-0.455(y^+ + 6)^{0.53}], \\ f_v &= 1 - \exp(-0.02y^+) \end{aligned} \tag{11}$$

for values of y^+ less than 80, approximately. The functions f_u and f_v defined above are strictly only valid for the particular low Reynolds number flow against which they are calibrated. However, even at higher Reynolds numbers it is known that the normal component of fluctuating velocity is over-estimated by the standard EIM approach and it is anticipated that predictions based on the above values of f_u and f_v will be better than those based on the assumption of isotropy. Also, the damping functions are strictly only applicable to the duct flow geometry for which they are derived, but they should be reasonably accurate for a wider range of geometries, including the wave-plate demister geometry for which they are intended to be used.

The other important issue in the EIM is the determination of the particle-eddy interaction time (T_i). Normally, the particle–eddy interaction time is chosen to be the minimum of the eddy lifetime and the time taken by a particle to cross an eddy. To facilitate the comparison with the results of Kallio and Reeks (1989), we assume that the particle–eddy interaction time T_i is identical to the eddy lifetime T_e, which is equivalent to assuming that no particles cross an eddy before the eddy dies. T_e is evaluated from the relationship

$$T_e = C_1 k/\epsilon \tag{12}$$

where ϵ is the rate of dissipation of turbulence kinetic energy, and C_1 is a constant. Kallio and Reeks (1989) sampled T_i from an exponential distribution whose mean value is the Lagrangian integral timescale whereas the choice of a constant eddy lifetime corresponds to a delta function distribution. Graham and James (1996) argued that whatever choice is made for the distribution, it should be ensured that the Lagrangian integral timescale obtained from simulations is correct for the idealised case of homogeneous, isotropic turbulence. This is assured for a constant eddy lifetime if C_1 is twice the value normally taken by other authors. However, we use the slightly larger value $C_1 = 0.53$ in the present work so that better agreement with the mean values obtained by Kallio and Reeks (1989) is achieved.

The calculation of particle dispersion is carried out in a duct flow with dimensionless half-width $h^+ = 180$ and dimensionless length $x^+ = 50{,}000$. The numerical particle tracking scheme is similar to that of Kallio and Reeks (1989) except for the method of the solution of Eq. (3). A small time step (0.2 dimensionless time units) is employed to enable the analytic expressions (5) and (6) to be used. As in Kallio and Reeks (1989), 20 equispaced bins across the channel are used to collect particles at different locations in the streamwise direction. The dimensionless particle deposition velocity is calculated by the same formula used by Kallio and Reeks (1989), i.e.

$$V_{\mathrm{d}}^{+}=\frac{U_{\mathrm{m}}A}{u_{\tau}P\delta x}\log(N_{\mathrm{in}}/N_{\mathrm{out}}) \tag{13}$$

where U_{m} is the bulk mean velocity, A and P are the area and perimeter of the duct, respectively, and δx is the length of the duct in which the number of particles entering the section, N_{in}, and the number of particles leaving the section, N_{out}, are counted.

3. Results

Predictions of particle normal rms velocity are made, when $\rho_{\mathrm{p}}/\rho_{\mathrm{f}}=590$, for $\tau^{+}=1.1$ and $\tau^{+}=14.3$, using the solution scheme described above and the data for the turbulent flow field given by Kallio and Reeks (1989). In total 2×10^{5} particles are tracked in the calculations (which take ~12 h cpu time on a DEC Alpha workstation). It can be seen from Fig. 1 that the results obtained in this paper agree well with those of Kallio and Reeks (1989).

Fig. 2 shows the dimensionless deposition velocity as a function of the particle relaxation time. It is seen that the results obtained with the unmodified EIM depart significantly from the data of Liu and Agarwal (1974) and from those obtained with the modified EIM. The discrepancy becomes significantly larger for particles with small relaxation times. This result demonstrates the unsuitability of the standard EIM for the flow considered. The dimensionless deposition velocities predicted with the exact DNS data and those using the damping functions

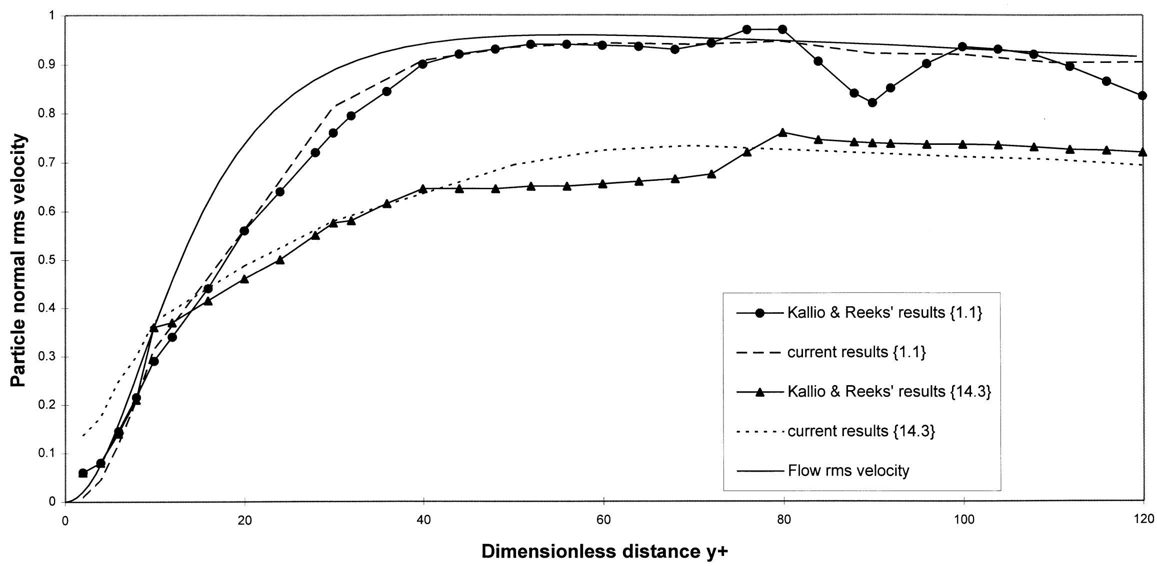

Fig. 1. Particle normal rms velocity predictions using Kallio and Reeks' data for the flow field. The number in braces denotes the value of τ^{+}.

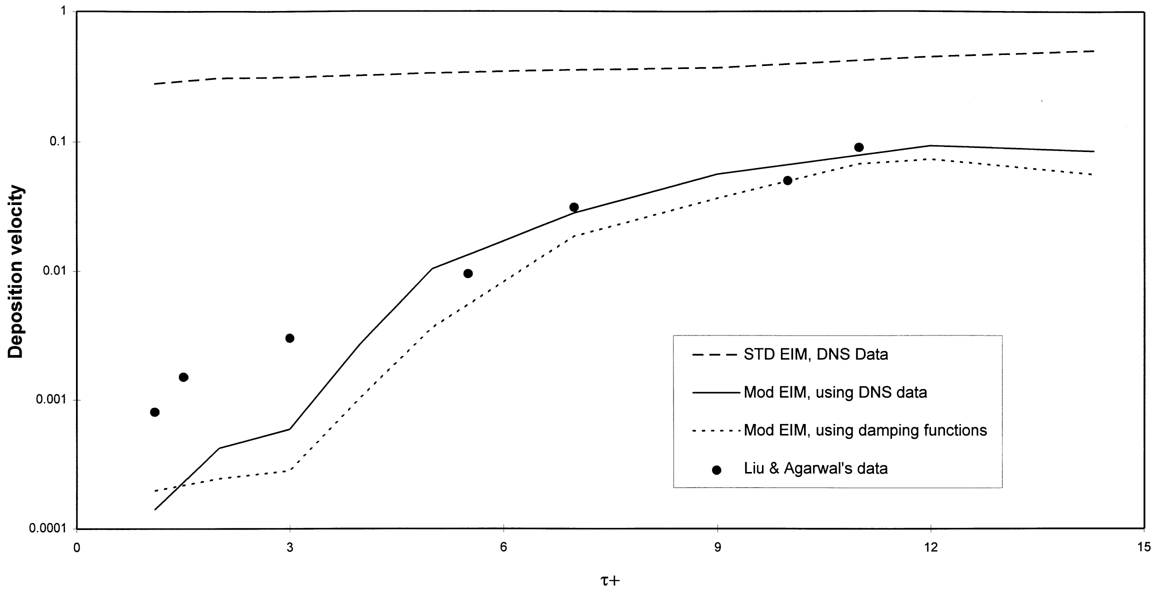

Fig. 2. The deposition velocity. STD EIM denotes the unmodified eddy interaction model results and Mod EIM those obtained with the modified EIM.

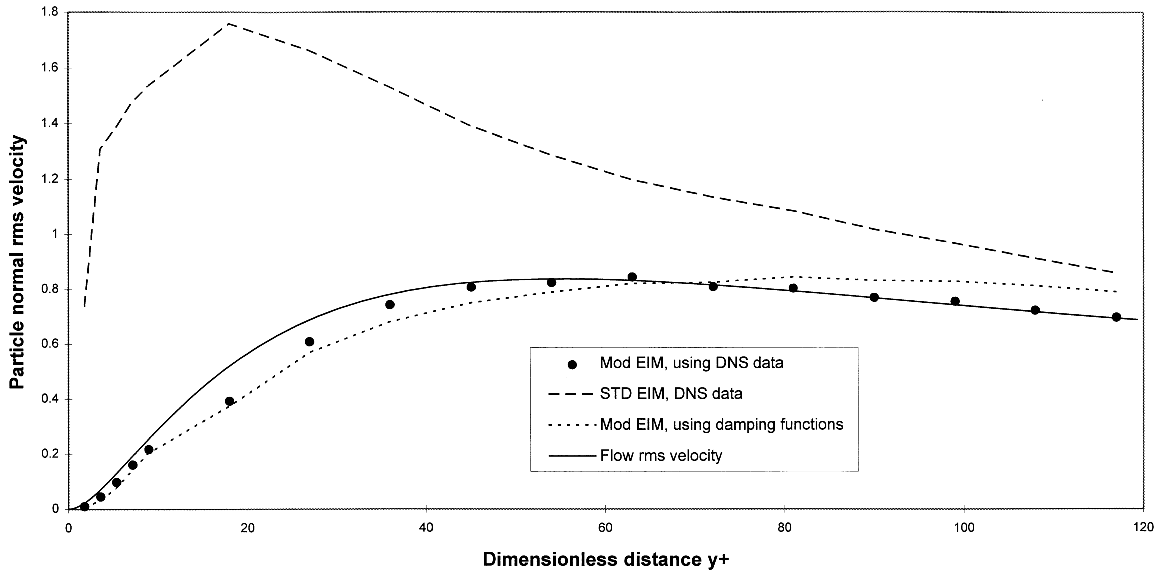

Fig. 3. Particle normal rms velocity, $\tau^+ = 1.1$. STD EIM denotes the unmodified eddy interaction model results and Mod EIM those obtained with the modified EIM.

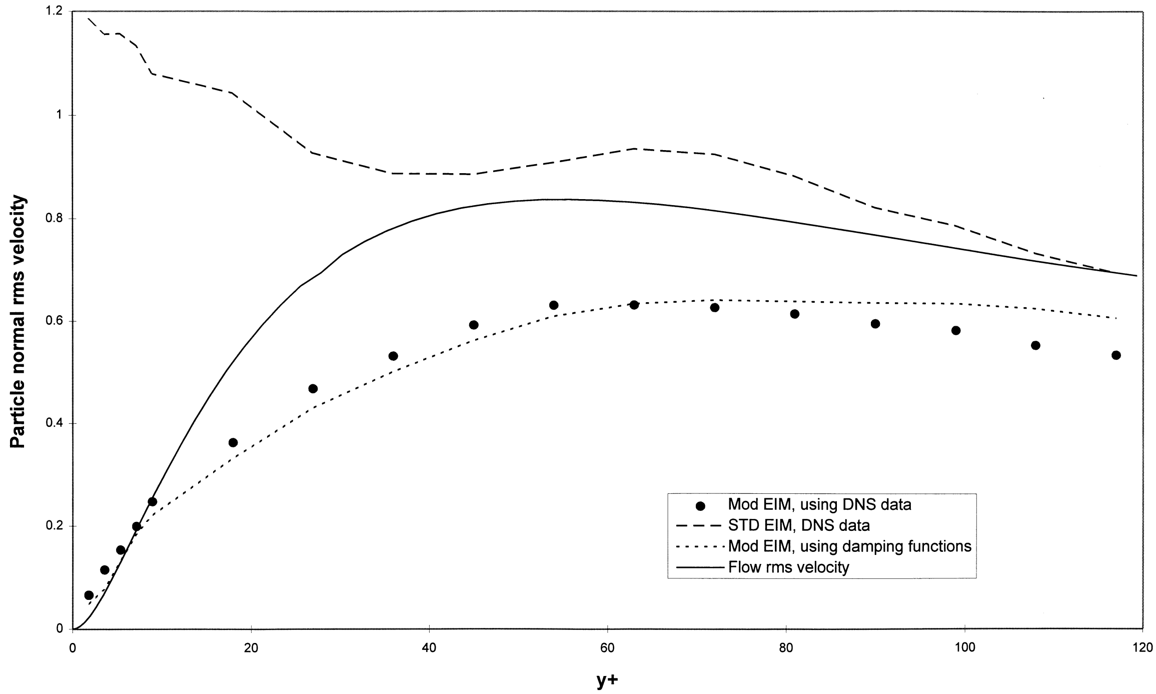

Fig. 4. Particle normal rms velocity, $\tau^+ = 14.3$. STD EIM denotes the unmodified eddy interaction model results and Mod EIM those obtained with the modified EIM.

agree with the measurements of Liu and Agarwal (1974) except for the region with very small particle relaxation times. The results of Kallio and Reeks (1989) suggest that this discrepancy is mainly due to the omission of the lift force.

Figs. 3 and 4 show the distributions of particle normal rms velocity across the duct for $\tau^+ = 1.1$ and $\tau^+ = 14.3$. It is also seen that results obtained with the standard EIM are unrealistic. The particle normal rms velocity is very close to that of the flow in the case of $\tau^+ = 1.1$. Comparison of Figs. 3 and 4 with Fig. 1 shows that particle normal rms velocities predicted using DNS data are smaller than those predicted using Kallio and Reeks' (1989) data for each of the cases $\tau^+ = 1.1$ and $\tau^+ = 14.3$. The turbulent flow data used by Kallio and Reeks (1989) are obtained by curve-fitting Laufer's (1954) experimental data, which correspond to pipe flows with Reynolds numbers of 5×10^4 and 5×10^5. The turbulent intensities of those pipe flows are much higher than those of the turbulent flow considered in this paper.

4. Conclusions

By using the DNS data of Kim et al. (1987) for a duct flow, and from comparison with the literature, it is shown that the combination of the standard EIM with an isotropic flow field yields unrealistic predictions for features of particle dispersion for particles with dimensionless relaxation times in the range 1.1–14.3. The simple damping method proposed in this paper can account for the influence of anisotropic turbulence in the near-wall region and it can be used within the framework of k–ϵ type turbulence models. The method is also efficient computationally. Results of particle dispersion obtained with the method agree well with the experimental data of Liu and Agarwal (1974). It is now intended to apply the modified method to the problem of small droplet deposition in wave–plate demisters.

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